第 29 卷 第 2 期　　湖南科技大学学报(自然科学版)　　Vol. 29 No. 2
2014 年　6 月　　Journal of Hunan University of Science & Technology(Natural Science Edition)　　Jun. 2014

doi:10.13582/j.cnki.1672-9102.2014.02.011

真空联合变堆载预压下竖井地基固结分析

郭彪[1,2],张发春[1],房锐[1],李果[1],陈贺[1]

(1. 云南省交通规划设计研究院 陆地交通气象灾害防治技术国家工程实验室,云南 昆明 650200;
2. 浙江大学 软弱土与环境土工教育部重点实验室,浙江 杭州 310058)

摘　要:为了得到真空联合变堆载预压下竖井地基沉降随时间的发展规律,考虑真空度沿竖井线性减小,堆载单级线性施加,还考虑了地基的径竖向渗流以及扰动区土体水平渗透系数呈抛物线变化,推导了真空联合变堆载预压下竖井地基固结度的一个解析解.并分析了地基固结性状.结果表明,荷载逐渐施加时,采用真空联合堆载预压比只采用堆载预压固结要快,真空度越大,沿深度衰减越慢,固结越快.在地基井径比和水平渗透系数与竖向渗透系数之比较小时,地基的竖向渗流对地基的固结度有较大的影响.

关键词:竖井地基;固结;真空联合堆载预压;渗透系数

中图分类号:TU47　　**文献标志码**:A　　**文章编号**:1672-9102(2014)02-0052-05

Consolidation analysis of vertical drain with vacuum and varying surcharge preloading

GUO Biao[1,2], ZHANG Fa-chun[1], FANG Rui[1], LI Guo[1], CHEN He[1]

(1. National Engineering Laboratory for Land Transport Meteorological Disaster Control Technology, Broadvision Engineering Consultants, Kunming 650200, China;
2. MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering, Zhejiang University, Hangzhou 310058, China)

Abstract: In order to get the settlement development pattern of vertical sand foundation under vacuum-surcharge preloading, A analytical solution was obtained for the consolidation of vertical sand foundation, under vacuum preloading considering, the vacuum degree was linearly changed along the vertical drain depth, the loading was applied in a single-stage way, both horizontal and vertical drainage, and the parabolic distribution pattern of horizontal permeability coefficient of soil was considered. Furthermore, the consolidation behavior of vertical drain foundation was analyzed. The results show that the loading is applied gradually, the larger the vacuum is, the faster the consolidation is, the slower the reducing of vacuum is, the faster the consolidation. Ignoring the vertical flow within the soil will under-estimate the consolidation rate. Furthermore, the less the radius ratio is, and the less the ratio of horizontal permeability to vertical permeability is, the greater the under-estimated value is.

Key words: vertical drains; consolidation; vacuum-surcharge preloading; permeability coefficient

真空预压法是由瑞典皇家地质学院 Kjellman[1] 1952 年提出的一种有效的软土地基处理方法,并可与堆载预压联合应用.要达到相同的处理效果,真空联合堆载预压可有效减小所需的堆载荷载;并且真空预压时土体的有效应力是等轴增大的,对应的侧向变形表现为向内压缩,因此可有效降低地基剪切破坏的风险.

真空预压下竖井地基的固结问题已有很多国内外学者作了研究.在早期的研究中,真空预压被当作均匀分布在地基表面的作用力,但试验表明,真空度会沿着竖井向地基深处传递,但真空度不断减小[2]. Indraratna et al[3-4] 考虑了这一因素,推导了

收稿日期:2013-05-08

通信作者:郭彪(1982-),男,四川安岳人,博士,高级工程师,主要从事地基处理及边坡治理研究工作. E-mail:gb25891775@163.com

只考虑地基径向渗流情况下地基的固结解析解. Rujikiatkamjorn[5-6]考虑了地基的径向和竖向组合渗流,但假设真空度沿竖井深度均匀分布,得到了解析解,并进行了数值分析. Walker R 等[7]用光谱法对固结进行了分析. Saowapakpiboon et al[8]对真空预压下竖井地基固结性状进行了试验研究. Geng, Saowapakpiboona et al[9-10]得到了变真空预压下竖井地基的固结数值解. Duong et al[11]等采用三维仪器对真空预压进行了模拟. 许忠发,熊熙等[12-13]对真空预压在路基处置中的应用进行了研究.

在目前的真空联合堆载研究中,均假设堆载是瞬时施加的. 但实际上堆载施加往往需要经过一段时间,所产生的附加应力也是随时间变化的.

在目前的真空联合堆载研究中,均认为涂抹区的渗透系数不变. 实际上,涂抹区土体的水平渗透性随离砂井的距离不同是逐渐变化的[14-15],离砂井越近,扰动越强,水平渗透系数越小,离砂井越远,扰动越弱,渗透性越接近天然土体.

针对以上情况,本文考虑真空度沿竖井线性下降,堆载线性施加,并考虑了地基的径竖向渗流以及扰动区土体水平渗透系数呈抛物线变化,推导了真空联合堆载预压下竖井地基固结度的解析解. 并分析了地基固结性状.

1　固结方程及求解条件

图 1 为真空联合堆载预压下砂井地基固结简化模型. 在本文的推导过程中,做了以下假定:

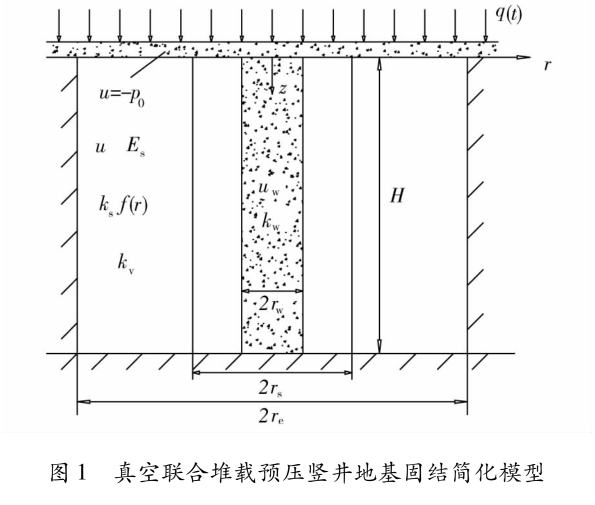


图 1　真空联合堆载预压竖井地基固结简化模型

1）土体是完全饱和的;

2）土颗粒和水都不可压缩,土体的变形完全由孔隙水的排出引起;

3）土体压缩模量以及竖向渗透系数保持不变;

4）土中水的渗流服从 Darcy 定律;

5）等应变条件成立;

6）真空负压 $-p(z)$ 瞬时施加,真空度沿竖井线性下降;即有

$$p(z) = p(0) - [p(0) - p(H)]\frac{z}{H}. \tag{1}$$

式中,$p(0)$,$p(H)$ 分别为砂井顶面和底面的真空度,kPa,$p(z)$ 为砂井深度 z 处的真空度,kPa.

7）堆载线性施加,在地基中引起的附加应力沿深度不变;即有

$$q(t) = \begin{cases} q_u \dfrac{t}{t_c}, t < t_c; \\ q_u, t \geqslant t_c. \end{cases} \tag{2}$$

式中,q_u 为最终荷载,kN,t_c 为达到最终荷载的时间,s.

根据等应变条件有

$$\frac{\partial \varepsilon_v}{\partial t} = \frac{\partial (q - \bar{u})}{E_s \partial t}. \tag{3}$$

式中,ε_v 为地基竖向应变;E_s 为土体压缩模量,MPa;$q(t)$ 为上部荷载,kN;$\bar{u}$ 为土体任一深度处的平均超静孔压,kPa,表达式为

$$\bar{u} = \frac{1}{\pi(r_e^2 - r_w^2)}\int_{r_w}^{r_e} 2\pi r u \mathrm{d}r. \tag{4}$$

式中,r_w 为砂井半径,m,r_e 为砂井影响区半径,m;u 为土体任一点的超静孔压 kPa.

砂井地基的固结方程为

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{k_m f(r)}{\gamma_w} r \frac{\partial u}{\partial r}\right] + \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2} = -\frac{\partial \varepsilon_v}{\partial t}. \tag{5}$$

式中,k_v 为土体竖向渗透系数,m/s;k_w 为砂井渗透系数,m/s;k_m 为最大水平渗透系数,m/s;$f(r)$ 为描述渗透系数随 r 变化的函数;$k_v f(r)$ 为土体水平向渗透系数,m/s.

本文考虑土体水平向渗透系数呈抛物线变化,如图 2 所示.

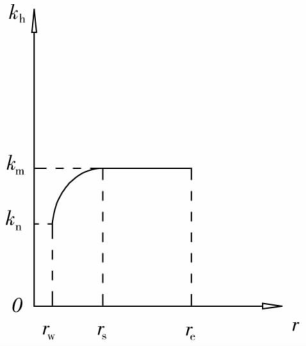


图 2　影响区土体水平渗透系数

根据图 2 可得到:

$$f(r) = \begin{cases} (1-\delta)\left(a - bs + b\dfrac{r}{r_w}\right)\left(a + bs - b\dfrac{r}{r_w}\right), r_w < r < r_s; \\ 1, \qquad r_s \leqslant r < r_e. \end{cases} \tag{6}$$

式中,$a = \sqrt{\dfrac{1}{(1-\delta)}}$, $b = \dfrac{1}{(s-1)}$;r_s 为涂抹区半径,m;δ 为最小渗透系数和最大渗透系数之比,0 <

$\delta < k_n / k_m < 1$.

径向边界条件为

$$r = r_e : \frac{\partial u}{\partial r} = 0. \tag{7}$$

$$r = r_w : u = -p(z) = -p(0) + [p(0) - p(H)]\frac{z}{H}. \tag{8}$$

对式(6)两边对 r 积分,并利用边界条件式(5)可得:

$$\frac{\partial u}{\partial r} = \frac{\gamma_w}{2k_m}\left(\frac{\partial \varepsilon_v}{\partial t} + \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2}\right)\left[\frac{r_e^2}{rf(r)} - \frac{r}{f(r)}\right]. \tag{9}$$

对式(9)两边对 r 积分,并利用边界条件式(8)可得:

$$u = \frac{\gamma_w}{2k_m}\left(\frac{\partial \varepsilon_v}{\partial t} + \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2}\right)[r_e^2 A_0(r) - B_0(r)] - p(0) + [p(0) - p(H)]\frac{z}{H}. \tag{10}$$

式中,

$$A_0(r) = \int_{r_w}^{r}\frac{\mathrm{d}x}{f(x)x}, B_0(r) = \int_{r_w}^{r}\frac{x\mathrm{d}x}{f(x)}. \tag{11}$$

将式(10)代入式(5)可得:

$$\bar{u} = \frac{\gamma_w}{(r_e^2 - r_w^2)k_m}\left(\frac{\partial \varepsilon_v}{\partial t} + \frac{k_v}{\gamma_w}\frac{\partial^2 \bar{u}}{\partial z^2}\right)(r_e^2 A_1 - B_1) - p(0) + [p(0) - p(H)]\frac{z}{H}. \tag{12}$$

式中,

$$A_1 = \int_{r_w}^{re} rA_0(r)\mathrm{d}r, B_1 = \int_{r_w}^{re} rB_0(r)\mathrm{d}r. \tag{13}$$

将式(4)代入式(12)并整理可得:

$$A\frac{\partial^2 \bar{u}}{\partial z^2} - B\frac{\partial \bar{u}}{\partial t} - \bar{u} = p(0) - [p(0) - p(H)]\frac{z}{H} - B\frac{\partial q}{\partial t}. \tag{14}$$

式中,

$$A = \frac{k_v r_e^2 F_a}{2k_m}, B = \frac{r_e^2 F_a}{2c_h}. \tag{15}$$

$$F_a = \frac{2(r_e^2 A_1 - B_1)}{r_e^2(r_e^2 - r_w^2)}, c_h = \frac{E_s k_m}{r_w}. \tag{16}$$

式(12)即为本文真空联合堆载预压下砂井地基固结问题的控制方程,其边界条件为

$$z = 0 : \bar{u} = -p(0). \tag{17}$$

$$z = H : \frac{\partial \bar{u}}{\partial z} = 0. \tag{18}$$

初始条件为

$$t = 0 : \bar{u} = q(0) = 0. \tag{19}$$

由式(6)可以得到 F_a 的表达式为

$$F_a = \frac{\alpha^2 F_a' + n^2 F_a''}{n^2 - 1}. \tag{20}$$

式中,

$$F'_a = \frac{1}{a^2 - b^2 s^2}\left(s^2 \ln s - \frac{s^2}{2} + \frac{1}{2}\right) - \frac{1}{a^2 b^2 - b^4 s^2}\left[\frac{1}{2} - bs + \left(\frac{a^2}{2} - b^2\right)\ln\delta + \frac{abcs}{2}\right] + \frac{1}{n^2 b^4}\left[\frac{1}{2} - 3bs + \left(\frac{a^2}{2} + b^2\right)\ln\delta + \frac{3abcs}{2}\right];$$

$$F''_a = a^2\left(1 - \frac{s^2}{n^2}\right)\left\{\frac{1}{a^2 - b^2}[\ln(s\sqrt{\delta}) -] - \frac{1}{n^2 b^2}\left(\ln\sqrt{\delta} + \frac{bcs}{2a}\right)\right\} + \ln\frac{n}{s} - \frac{3}{4} + \frac{s^2}{n^2} - \frac{s^4}{4n^4};$$

$$c = \ln\left[\frac{(a + 1)}{(a - 1)}\right].$$

2 方程求解

设控制方程式(14)的解为

$$\bar{u}(z,t) = -p(0) + \sum_{m=1}^{\infty} T_m(t)\sin\left(\frac{M}{H}z\right). \tag{21}$$

式中,$M = (2m - 1)\pi/2$, $m = 1,2,3\cdots\cdots$

式(21)已经满足边界条件式(17)和式(18),将其代入初始条件式(19),有

$$-p(0) + \sum_{m=1}^{\infty} T_m(0)\sin\left(\frac{M}{H}z\right) = 0. \tag{22}$$

将式(22)两边同乘以 $\sin(Mz/H)$ 并在$[0, H]$上对 z 积分,并利用三角级数的正交性可得:

$$T_m(0) = \frac{2}{M}p(0). \tag{23}$$

将式(21)代入式(14),整理后可得:

$$A\left(\frac{M}{H}\right)^2 \sum_{m=1}^{\infty} T_m(t)\sin\left(\frac{M}{H}z\right) + B\sum_{m=1}^{\infty} T'_m(t)\sin\left(\frac{M}{H}z\right) + \sum_{m=1}^{\infty} T_m(t)\sin\left(\frac{M}{H}z\right) = p(0) - p(z) + B\frac{\partial q}{\partial t}. \tag{24}$$

同样将式(24)两边同乘以 $\sin(Mz/H)$ 并在$[0, H]$上对 z 积分,并利用三角级数的正交性可得:

$$T'_m(t) + \beta_m T_m(t) = Q_m(t). \tag{25}$$

式中,

$$Q_m(t) = \frac{2}{BH}\int_0^H\left[p(0) - p(z) + B\frac{\partial q}{\partial t}\right]\sin\left(\frac{M}{H}z\right)\mathrm{d}z. \tag{26}$$

$$\beta_m = \frac{1}{B} + \frac{A}{B}\left(\frac{M}{H}\right)^2. \tag{27}$$

式(25)为关于 $T_m(t)$ 的一阶线性微分方程,式(23)为其定解条件,其解可写为

$$T_m(t) = \mathrm{e}^{-\beta_m t}\left[\int_0^t Q_m(\tau)\mathrm{e}^{\beta_m \tau}\mathrm{d}\tau + \frac{2}{M}p(0)\right]. \tag{28}$$

将式(26)代入式(28)可得:

$$T_m(t) = \frac{2\mathrm{e}^{-\beta_m t}}{BH}\left\{\int_0^t \mathrm{e}^{\beta_m \tau}\int_0^H\left[p(0) - p(z) + B\frac{\partial q(\tau)}{\partial \tau}\right]\sin\left(\frac{M}{H}z\right)\mathrm{d}z\mathrm{d}\tau + Bp(0)\frac{H}{M}\right\}. \tag{29}$$

将式(1)、式(2)代入式(29)后再代入式(21)可得:

当 $t < t_c$ 时,

$$\begin{aligned}\bar{u}(z,t) = & -p(0) + \sum_{m=1}^{\infty} \frac{2\mathrm{e}^{-\beta_m t}}{BH} \\ & \sin\left(\frac{M}{H}z\right)\left\{\left[(-1)^{m+1}\frac{p(0)-p(H)}{M^2}H + \frac{BHq_{\mathrm{u}}}{t_{\mathrm{c}}M}\right]\cdot \right. \\ & \left. \frac{1}{\beta_m}(\mathrm{e}^{\beta_m t}-1) + Bp(0)\frac{H}{M}\right\}.\end{aligned} \tag{30}$$

当 $t \geqslant t_{\mathrm{c}}$ 时，

$$\begin{aligned}\bar{u}(z,t) = & -p(0) + \sum_{m=1}^{\infty} \frac{2}{BH}\sin\left(\frac{M}{H}z\right)\left\{(-1)^{m+1}\cdot \right. \\ & \frac{p(0)-p(H)}{M^2}\frac{H}{\beta_m}(1-\mathrm{e}^{-\beta_m t}) + \frac{BHq_{\mathrm{u}}}{M\beta_{\mathrm{m}}t_{\mathrm{c}}} \\ & \left. [\mathrm{e}^{-\beta_m(t-t_{\mathrm{c}})} - \mathrm{e}^{-\beta_m t}] + Bp(0)\frac{H}{M}\mathrm{e}^{-\beta_m t}\right\}.\end{aligned} \tag{31}$$

由式(10)、式(12)可得地基任意一点孔压为

$$u(z,r,t) = \frac{r_{\mathrm{e}}^2A_0(r) - B_0(r)}{(r_{\mathrm{e}}^2 - r_{\mathrm{w}}^2)F_{\mathrm{a}}}[\bar{u} + p(z)] - p(z). \tag{32}$$

将式(1)、式(30)、式(31)代入上式可得：

当 $t < t_{\mathrm{c}}$ 时，

$$\begin{aligned}u(z,r,t) = & [p(0)-p(H)]\frac{z}{H} - p(0) + \frac{r_{\mathrm{e}}^2A_0(r) - B_0(r)}{(r_{\mathrm{e}}^2 - r_{\mathrm{w}}^2)F_{\mathrm{a}}} \\ & \left\{-[p(0)-p(H)]\frac{z}{H} + \sum_{m=1}^{\infty}\frac{2}{BH}\cdot \right. \\ & \sin\left(\frac{M}{H}z\right)\left\{\left[(-1)^{m+1}\frac{p(0)-p(H)}{M^2}H + \frac{BHq_{\mathrm{u}}}{t_{\mathrm{c}}M}\right] \right. \\ & \left.\left. \frac{1}{\beta_m}(1-\mathrm{e}^{-\beta_m t}) + Bp(0)\frac{H}{M}\mathrm{e}^{-\beta_m t}\right\}\right\}.\end{aligned} \tag{33}$$

当 $t \geqslant t_{\mathrm{c}}$ 时，

$$\begin{aligned}u(z,r,t) = & [p(0)-p(H)]\frac{z}{H} - p(0) + \\ & \frac{r_{\mathrm{e}}^2A_0(r) - B_0(r)}{(r_{\mathrm{e}}^2 - r_{\mathrm{w}}^2)F_{\mathrm{a}}}\left\{-[p(0)-p(H)]\frac{z}{H} + \sum_{m=1}^{\infty}\frac{2}{BH}\cdot \right. \\ & \sin\left(\frac{M}{H}z\right)\left\{(-1)^{m+1}\frac{p(0)-p(H)}{M^2}\frac{H}{\beta_m}(1-\mathrm{e}^{-\beta_m t}) + \right. \\ & \left.\left. Bp(0)\frac{H}{M}\mathrm{e}^{-\beta_m t} + \frac{BHq_{\mathrm{u}}}{M\beta_m t_{\mathrm{c}}}[\mathrm{e}^{-\beta_m(t-t_{\mathrm{c}})} - \mathrm{e}^{-\beta_m t}]\right\}\right\}.\end{aligned} \tag{34}$$

地基总平均固结度可表示为

$$U = \frac{S_t}{S_\infty} \tag{35}$$

式中，S_t 为 t 时刻地基的沉降，m，S_∞ 为地基的最终沉降，m.

$$S_t = \int_0^H \varepsilon_{\mathrm{v}}\mathrm{d}z = \frac{1}{E_{\mathrm{s}}}\int_0^H (q - \bar{u})\mathrm{d}z. \tag{36}$$

令 $t \to \infty$，有 $q \to q_{\mathrm{u}}$，$\bar{u}(z,t) \to -p(z)$，可得地基最终沉降为

$$S_\infty = \frac{1}{E_{\mathrm{s}}}\int_0^H [q_{\mathrm{u}} + p(z)]\mathrm{d}z \tag{37}$$

于是可得变形定义的总平均固结度为

当 $t < t_{\mathrm{c}}$ 时，

$$\begin{aligned}U = & \frac{2}{2q_{\mathrm{u}} + p(0) + p(H)}\left\{\frac{t}{t_{\mathrm{c}}}q_{\mathrm{u}} + p(0) - \right. \\ & \sum_{m=1}^{\infty}\frac{2}{BM}\left\{\left[(-1)^{m+1}\frac{p(0)-p(H)}{M^2} + \frac{Bq_{\mathrm{u}}}{t_{\mathrm{c}}M}\right]\cdot \right. \\ & \left.\left. \frac{1}{\beta_m}(1-\mathrm{e}^{-\beta_m t}) + \frac{Bp(0)}{M}\mathrm{e}^{-\beta_m t}\right\}\right\}.\end{aligned} \tag{38}$$

当 $t \geqslant t_{\mathrm{c}}$ 时，

$$\begin{aligned}U = & \frac{2}{2q_{\mathrm{u}} + p(0) + p(H)}\left\{q_{\mathrm{u}} + p(0) - \right. \\ & \sum_{m=1}^{\infty}\frac{2}{BM}\left\{(-1)^{m+1}\frac{p(0)-p(H)}{M^2\beta_m}(1-\mathrm{e}^{-\beta_m t}) + \right. \\ & \left.\left. \frac{Bp(0)}{M}\mathrm{e}^{-\beta_m t} + \frac{Bq_{\mathrm{u}}}{M\beta_m t_{\mathrm{c}}}[\mathrm{e}^{-\beta_m(t-t_{\mathrm{c}})} - \mathrm{e}^{-\beta_m t}]\right\}\right\}.\end{aligned} \tag{39}$$

3　地基固结性状分析

图 3 是不同因素对固结度的影响，图中的参数取值为：$n=10$，$s=5$，$H/r_{\mathrm{e}}=20$，$k_{\mathrm{m}}/k_{\mathrm{v}}=2$，$\delta=0.4$.

图 3(a)为加荷历时对砂井地基固结的影响. 可以看出，在最终荷载相同的情况下，达到最终荷载的时间越长，固结越慢. 当 $T_{\mathrm{hc}}=0$ 时，为荷载瞬时施加，固结最快. T_{hc} 为地基的水平固结时间因子.

图 3(b)是真空度对砂井地基固结的影响. 可以看出，真空度越大，砂井地基固结越快.

图 3(c)是真空度沿深度的衰减数速度对竖井地基固结的影响. 可以看出，真空度衰减越慢，砂井地基固结越快.

图 3(d)是地基深度与砂井影响区半径之比 H/r_{e} 对砂井地基固结度的影响. 从图中可以看出，H/r_{e} 越小，地基固结越快，与不考虑地基竖向渗流时的差别越大.

图 3(e)是地基径竖向渗透系数之比 $k_{\mathrm{m}}/k_{\mathrm{v}}$ 对砂井地基固结度的影响. 可以看出，在 k_{m} 不变的情况下，k_{v} 越大，地基固结越快，与不考虑地基竖向渗流时的差别越大.

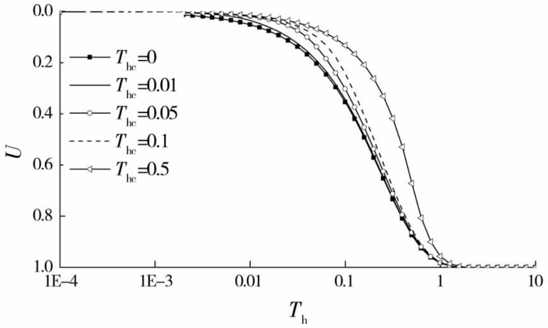
4　结论

本文考虑真空度沿竖井线性减小，同时考虑堆载为单级线性施加，还考虑了地基的径竖向渗流以及扰动区土体水平渗透系数呈抛物线变化，推导了真空联合堆载预压下竖井地基固结度的一个解析解. 并分析了地基固结性状. 分析结果表明：

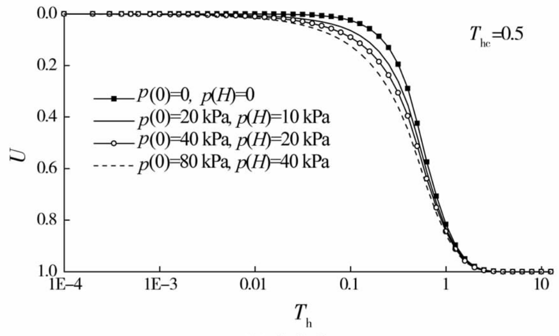
1）在最终荷载相同的情况下，达到最终荷载的时间越长，固结越慢.

2）采用真空联合堆载预压比只采用堆载预压固结要快，真空度越大，沿深度衰减越慢，固结越快.

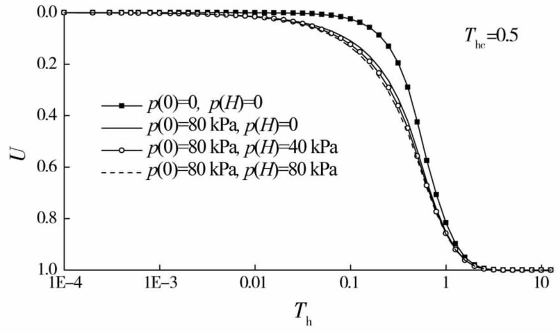
3）在地基 H/r_{e} 和 $k_{\mathrm{m}}/k_{\mathrm{v}}$ 较小时，地基的竖向渗流对地基的固结度有较大的影响.



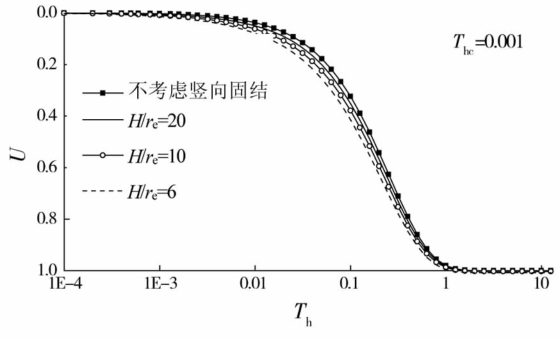
(a)加荷历时



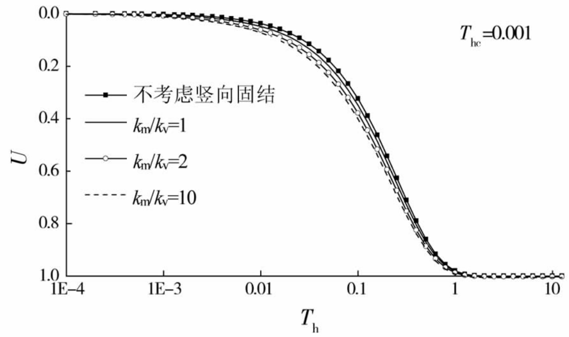
(b)真空度



(c)真空度下降率



(d) H/r_e



(e) k_m/k_v

图3 不同因素对砂井地基固结度的影响

参考文献:

[1] Kiellman W. Consolidation of clayey soils by atmospheric pressure [C]//Proceedings of a Conference on Soil Stabilization. Boston: Massachusetts Institute of Technology Press, 1952.

[2] Chu J, Yan S W, Yang H. Soil improvement by the vacuum preloading method for an oil storage station[J]. Geotechnique, 2000, 50(6):625－632.

[3] Indraratna B, Bamunawita C, Khabbaz H. Numerical modeling of vacuum preloading and field applications[J]. Canadian Geotechnical Journal, 2004, 41(6): 1098－1110.

[4] Indraratna B, Sathananthan I, Rujikiatkamjorn C, et al. Aanlytical and numerical modeling of soft soil stabilized by PVD incorporating vacuum preloading[J]. International Journal of Geomechanics, 2005, 5(2): 114－124.

[5] Rujikiatkamjorn C, Indraratna B. Analytical solutions and design curves for vacuum－assisted consolidation with both vertical and horizontal drainage [J]. Canadian Geotechnical Journal, 2007(44): 188－200.

[6] Rujikiatkamjorn C, Indraratna B, Chu J. 2D and 3D numerical modeling of combined surcharge and vacuum preloading with vertical drains[J]. International Journal of Geomechanics, 2008, 8 (2):144－156.

[7] Walker R, Indraratna B. Consolidation analysis of a stratified soil with vertical and horizontal drainage using the spectral method[J]. Geotechnique, 2009, 59(5): 439－449.

[8] Saowapakpiboon J, Bergado D T, Youwai S, et al. Measured and predicted performance of prefabricated vertical drains (PVDs) with and without vacuum preloading[J]. Geotextiles and Geomembranes, 2010, 28 (1): 1－11.

[9] Geng X Y, Indraratna B, Rujikiatkamjorn C. Analytical solutions for a single vertical drain with timedependent vacuum combined surcharge preloading in membrane and membraneless systems[J]. IOP Conference Series: Materials Science and Engineering, 2010,10(1):012117－1－012117－8.

[10] Saowapakpiboona J, Bergado D T, Voottipruex P, et al. PVD improvement combined with surcharge and vacuum preloading including simulations [J]. Geotextiles and Geomembranes, 2011, 29(1):74－82.

[11] Duong N T, Teparaksa W, Tanaka H. Simulation vacuum preloading method by Tri－Axial apparatus[J]. International Journal of Geosciences, 2012, 3(1):211－221.

[12] 许忠发, 何钜. 真空－堆载联合预压法在高速公路软基加固中的应用研究[J]. 水利与建筑工程学报, 2012, 42(2): 165－170.

[13] 熊熙, 王杰光, 曹辉, 等. 真空联合堆载预压与堆载预压软土地基加固对比研究[J]. 路基工程, 2012(3):53－56.

[14] Hawlader B C, Imai G, Muhunthan B. Numerical study of the factors affecting the consolidation of clay with vertical drains [J]. Geotextiles and Geomembranes, 2002, 20(4): 213－239.

[15] 卢萌盟, 谢康和, 张玉国, 等. 考虑土体水平渗透系数变化的复合地基固结解[J]. 浙江大学学报(工学版), 2008, 41(11): 1996－2001.